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Note on generalized Janus configurations

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Abstract: We study several aspects of generalized Janus configuration, which includes a theta term. We investigate the vacuum structure of the theory and find that unlike the Janus configuration without theta term there is no nontrivial vacuum. We also discuss BPS soliton configuration both by supersymmetry analysis and from energy functional. The half BPS configurations could be realized by introducing transverse (p,q)-strings in original brane configuration corresponding to generalized Janus configuration. It turns out the BPS soliton could be taken as modified dyon. We discuss the solution of half BPS equations for the sharp interface case. Moreover we construct less supersymmetric Janus configuration with theta term.

Keywords: Supersymmetry and Duality, Brane Dynamics in Gauge Theories, p-branes, Solitons Monopoles and Instantons.

Contents

1. Introduction

Janus configuration is a kind of field theory with spatially dependent coupling constant. Its discovery was motivated by the attempt of generalizing AdS/CFT correspondence [1] to the case with spatially varying dilaton. The original Janus supergravity solutions is an one-parameter family solutions of dilatonic deformation of AdS_5, which breaks all supersymmetry but preserves full R-symmetry [2]. Later on , the other cases preserving part of R-symmetry and supersymmetry were discovered [3 − 5]. Especially, the half BPS Janus solutions with global symmetry $OSP(4|4)$ were discovered [6 − 8]. The dual gauge theories of the Janus solutions are supersymmetric deformations of $N = 4$ super Yang-Mills theory with coupling constant depending on one spatial coordinate [3, 9].[1] Some properties of half supersymmeric Janus Yang-Mills theory including vacuum structure and BPS configuration were discussed in [11]. For the vacuum structure, it was found that except for the ordinary Coulomb phase where the scalars are homogenous and diagonal, there are additional vacuum structure characterized by Nahm equations. The BPS configurations of supersymmetric Janus field theory include 1/2 BPS magnetic monopole and 1/4 BPS dyonic monopole.

However all of the above Janus field theory do not contain a theta term. This sounds strange since one can obtain Janus solution in Type IIB supergravity with both dilaton and axion through an $\mathrm{SL}(2, R)$ transformation. Recently, this puzzle was solved. In [12], the authors obtained generalized Janus configuration with spatially varying theta angle. The global symmetry is still $OSP(4|4)$ but its embedding in $PSU(4|4)$ is inequivalent to the one in Janus configuration without θ-term. In order to make the θ depend on one

[1] For a field theory with coupling constant depending on a lightlike coordinates and its AdS/CFT correspondence, see [10].

spatial coordinate, the embedding must also depend on the spatial coordinate. On the other hand, Janus configuration is closely related to the field theory with boundary [13, 14]. This could be seen from the realization of half-supersymmetric Janus by brane configuration. The corresponding brane configuration is N D3-branes ending on k successive five branes. The limit that k become large and the five-branes are closely spaced corresponds to the generalized Janus configuration with arbitrary y-dependent ψ, which is related to the Yang-Mills coupling $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2} = a + 4\pi D e^{2i\psi}$ with a and D being constants. Actually in the brane realization, the a and D could be determined by the brane configurations [12]. For example, for the configuration of D3's ending on NS5 branes one has $a = -4\pi D$. However there are no such constraints in Janus field theory. In this sense, Janus field theory seems to be more general than the brane configuration. However such brane configuration may help us to understand physics of Janus configuration.

In this paper, we try to study several aspects of the Janus configurations with theta term. We first study its moduli space and BPS configurations. We show that unlike the Janus configuration without theta term there is no nontrivial vacuum. We obtain the BPS soliton equations by imposing more projective conditions on the supersymmetry parameter in the field theory. We figure out the brane picture corresponding to half BPS soliton, which requires the introduction of (p,q)-string extending along one of transverse directions of D3-branes. It turns out that the half BPS soliton could be taken as modified dyon and 1/4-BPS soliton could be taken as modified string junction. We discuss the half-BPS solution in the sharp interface case. Finally we turn to construct generalized Janus configuration with less supersymmetries and find that there is no nontrivial vacuum.

The organization of this paper is as follows. In section 2, we give a brief review of Janus field theory including a spatial varying theta term, and then discuss its vacuum structure. This will help us to set up our convention. In section 3, we study the BPS solutions of generalized Janus configuration and their corresponding brane configurations. In section 4, we construct the less supersymmetric Janus field theory. We end with some conclusions and discussions.

2. Janus configuration with theta-angle and its vacuum

The four dimensional $N = 4$ supersymmetric Yang-Mills theory allows a deformation which result in a field theory with a spatially-dependent coupling constant and half supersymmetries [3, 9]. The deformed field theory is called Janus configuration. The Janus configuration has been extended to include a spatially varying θ angle [12]. We start from a brief review of this so-called generalized Janus configuration. The unperturbed $N = 4$ supersymmetric Yang-Mills theory could be written in ten-dimensional notation

$$I = \int d^4x \frac{1}{e^2} \mathrm{Tr} \left(\frac{1}{2} F_{IJ} F^{IJ} - i\overline{\Psi}\Gamma^I D_I \Psi \right) \tag{2.1}$$

and the supersymmetry transformations are

$$\delta_0 A_I = i\bar{\epsilon}\Gamma_I \Psi \tag{2.2}$$

$$\delta_0 \Psi = -\frac{1}{2}\Gamma^{IJ} F_{IJ}\epsilon \tag{2.3}$$

where $I, J = 0, 1, 2, \cdots, 9$, $A_I = A_I^a T_a, \mathrm{Tr}(T_a T_b) = -\delta_{ab}/2$, $T_a^\dagger = -T_a$. The gamma matrices Γ^I's are in Majorana representation, where $\{\Gamma^I, \Gamma^J\} = 2g^{IJ}$ with signature $(- + + \ldots +)$. The gaugino field Ψ and the supersymmetry transformation parameter ϵ are Majorana-Weyl spinors, obeying $\overline{\Gamma}\epsilon = \epsilon, \overline{\Gamma}\Psi = \Psi$, where $\overline{\Gamma} = \Gamma_{012\ldots 9}$. All the fields are defined in four-dimensional spacetime x^0, x^1, x^2, x^3. Without losing generality, we pick up one spacial coordinate $y = x^3$ on which the coupling constant and θ-angle depend. The components of 10-dimensional gauge fields A_I with $I = 4, 5 \ldots 9$ correspond to six four-dimensional scalar fields X_I. In this case, the notations $F_{IJ}, F_{\mu I}$ mean that $F_{IJ} = [X_I, X_J], F_{\mu I} = D_\mu X_I$ when $I, J = 4, 5 \ldots 9$. For simplicity, we use the ten-dimensional notations F_{IJ} in the above sense. The theta term is

$$
\begin{aligned}
I_\theta &= -\frac{1}{32\pi^2}\int d^4x\theta(y)\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}F_{\mu\nu}F_{\alpha\beta} \\
&= \frac{1}{8\pi^2}\int d^3x dy \frac{d\theta}{dy}\varepsilon^{\mu\nu\lambda}\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda\right). \qquad (2.4)
\end{aligned}
$$

where $\varepsilon^{0123} = -1, \varepsilon^{012} = 1$

The original Janus configurations without theta angle which have eight supercharges is given by modifying the action and supersymmetry transformation such that half of supersymmetries are preserved. In this case, the supersymmetry transformation parameter ϵ does not depend on y. However, ϵ should depend on y in order to add the theta term. Since the supersymmetry transformation is global, any two unbroken supersymmetry transformations should close into a translation along x^0, x^1, x^2. This requires

$$
\frac{d}{dy}\bar{\epsilon}\Gamma^\mu\tilde{\epsilon} = 0,\ \ \mu = 0, 1, 2, \quad \bar{\epsilon}\Gamma^3\tilde{\epsilon} = 0. \qquad (2.5)
$$

There are also other global symmetries. The original symmetry SO(1, 9) is broken to $W = \mathrm{SO}(1,2) \times \mathrm{SO}(3)_X \times \mathrm{SO}(3)_Y$. The SO(1, 2) is the Lorentz symmetry of spacetime in x^0, x^1, x^2. The $\mathrm{SO}(3)_X$ acts on X_4, X_5, X_6 while the $\mathrm{SO}(3)_Y$ acts on X_7, X_8, X_9. Taking both the global symmetries and dimension analysis into account, we can make the following ansatz for the modified supersymmetry and action transformation.

$$
\begin{aligned}
\delta_1\Psi &= \frac{-1}{2}\left(\Gamma^a X_a(s_1\Gamma_{456} + s_2\Gamma_{789}) + \Gamma^p X_p(t_1\Gamma_{456} + t_2\Gamma_{789})\right)\epsilon \qquad (2.6) \\
I' &= \int d^4x\frac{i}{e^2}\mathrm{Tr}\overline{\Psi}\left(\alpha\Gamma_{012} + \beta\Gamma_{456} + \gamma\Gamma_{789}\right)\Psi \qquad (2.7) \\
I'' &= \int d^4x\frac{1}{e^2}\left(u\varepsilon^{\mu\nu\lambda}\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda\right)\right. \\
&\qquad\left. + \frac{v}{3}\varepsilon^{abc}\mathrm{Tr}X_a[X_b, X_c] + \frac{w}{3}\varepsilon^{pqr}\mathrm{Tr}X_p[X_q, X_r]\right) \qquad (2.8) \\
I''' &= \int d^4x\mathrm{Tr}\left(\frac{r}{2e^2}X_a X^a + \frac{\tilde{r}}{2e^2}X_p X^p\right) \qquad (2.9)
\end{aligned}
$$

where $\mu, \nu, \lambda = 0, 1, 2, a, b, c = 4, 5, 6, p, q, r = 7, 8, 9$ and ε are antisymmetric tensors normalized to $\varepsilon^{012} = \varepsilon^{456} = \varepsilon^{789} = 1$. All parameters depend on y. Using the condition that

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the Lagrangian preserves half of supersymmetries, we can obtain the following equations:

$$\frac{d\epsilon}{dy} = \alpha\Gamma_{0123}\epsilon \tag{2.10}$$

$$\bar{\epsilon}((s_1+2\beta)B_1+(s_2-2\gamma)B_2+q) = 0 \tag{2.11}$$

$$\bar{\epsilon}((t_1-2\beta)B_1+(t_2+2\gamma)B_2+q) = 0 \tag{2.12}$$

$$\bar{\epsilon}(4\alpha B_0+2\beta B_1+2\gamma B_2-q) = 0 \tag{2.13}$$

$$u = -4\alpha, \quad v = -4\beta, \quad w = -4\gamma \tag{2.14}$$

$$\bar{\epsilon}((-2\beta'-4\gamma\alpha)B_1+(2\gamma'-4\beta\alpha)B_2) = \bar{\epsilon}\lambda \tag{2.15}$$

$$r = \lambda+2\beta^2+2\gamma^2-\frac{q^2}{2}-q' \tag{2.16}$$

$$\tilde{r} = -\lambda+2\beta^2+2\gamma^2-\frac{q^2}{2}-q', \tag{2.17}$$

where

$$B_0 = \Gamma_{456789}, B_1 = \Gamma_{3456}, B_2 = \Gamma_{3789}, \tag{2.18}$$

$$q = e^2\frac{d}{dy}\frac{1}{e^2} \tag{2.19}$$

and $'$ means d/dy. One can take $q, d\epsilon/dy$ and the parameters $\alpha,\beta,\gamma,s_i,t_j,u,v,w$ to be of first order, while $r,\tilde{r}$, the second derivatives of e^2,ϵ, the first derivatives of the other parameters, and the quadratic expressions of the first order quantities to be of second order. Note that the first five equations come from the conditions that the first order variation of action under supersymmetry transformation vanish, while the vanishing of the second order variation leads to the last three equations. Introducing another parameter ψ, we set $\psi' = 2\alpha$. We can solve the equation (2.10)

$$\epsilon = \left(\cos\frac{\psi}{2}-\sin\frac{\psi}{2}B_0\right)\epsilon_0, \quad B_2\epsilon_0 = \epsilon_0, \tag{2.20}$$

where ϵ_0 is a constant spinor. The solution is equivalent to impose the following projection condition

$$(\sin\psi B_1+\cos\psi B_2)\epsilon = \epsilon, \tag{2.21}$$

which is the projection condition on ϵ representing half of supersymmetries. The ϵ also satisfy the requirement of closure of supersymmetry (2.5) if $\bar{\epsilon}_0\Gamma^3\tilde{\epsilon}_0 = 0$. The equations from (2.11) to (2.15) are equivalent to the projection condition (2.21). So it is easy to obtain the following results

$$\psi' = 2\alpha, \qquad \beta = -\frac{\psi'}{2\cos\psi}, \quad \gamma = \frac{\psi'}{2\sin\psi}, \qquad \frac{1}{e^2} = D\sin 2\psi \tag{2.22}$$

$$u = -4\alpha, \qquad v = -4\beta, \qquad w = -4\gamma, \qquad \theta = 2\pi a+8\pi^2 D\cos 2\psi \tag{2.23}$$

$$s_1 = 2\psi'\frac{\sin^2\psi}{\cos\psi}, \quad s_2 = 2\psi'\sin\psi, \quad t_1 = -2\psi'\cos\psi, \quad t_2 = -2\psi'\frac{\cos^2\psi}{\sin\psi}. \tag{2.24}$$

Since θ and $1/e^2$ can be expressed in terms of the usual complex coupling parameter $\tau = \frac{\theta}{2\pi}+\frac{4\pi i}{e^2}$ which takes values in the upper half plane, we have $\tau = a+4\pi De^{2i\psi}$.

To obtain the vacuum structure of generalized Janus configurations, one may find the moduli space preserving full supersymmetries. However, the supersymmetry transformation parameter ϵ depends on y. It turns out to be more convenient to use the constant spinor ϵ_0. All of the projection conditions on ϵ can be reexpressed in terms of the projection conditions on the constant spinor ϵ_0. We can use ϵ_0 as the supersymmetry transformation parameter instead of ϵ. The total supersymmetry transformation on gaugino is

$$\delta\Psi = \frac{-1}{2}\left(\Gamma_{IJ}F^{IJ} + \Gamma^a X_a(s_1\Gamma_{456} + s_2\Gamma_{789}) + \Gamma^p X_p(t_1\Gamma_{456} + t_2\Gamma_{789})\right)\epsilon. \tag{2.25}$$

Note that after using (2.24) and the projection condition (2.21), we have

$$\begin{aligned}\frac{1}{2}\left(\Gamma^a X_a\left(s_1\Gamma_{456} + s_2\Gamma_{789}\right)\right)\epsilon &= \Gamma^{3a}X_a\frac{(\cos\psi)'}{\cos\psi}\epsilon\\ \frac{1}{2}\left(\Gamma^p X_p(t_1\Gamma_{456} + t_2\Gamma_{789})\right)\epsilon &= \Gamma^{3p}X_p\frac{(\sin\psi)'}{\sin\psi}\epsilon,\end{aligned}$$

where $a = 4, 5, 6,\quad p = 7, 8, 9$. In order to make the expression simple, we denote

$$\overline{F}_{3a} = \frac{D_3(X_a\cos\psi)}{\cos\psi},\qquad \overline{F}_{3p} = \frac{D_3(X_p\sin\psi)}{\sin\psi}. \tag{2.26}$$

Let us consider the moduli space of the theory. We can take $A_\mu = 0$ and six real scalars only depend on x_3. The vacuum configurations preserve all supersymmetries. With the above ansatz, the gaugino transformation (2.25) becomes

$$\begin{aligned}\delta\Psi &= -e^{-\psi B_0/2}\Big(\left(-(X_a\cos\psi)'\Sigma^a + (X_p\sin\psi)'\Sigma^p - [X_a, X_p]\Sigma^a\Sigma^p\right)B_0\epsilon_0\\ &\quad+\Big(\tan\psi(X_a\cos\psi)'\Sigma^a + \epsilon^{abc}[X_b, X_c]\Sigma^a/2 + \cot\psi(X_p\sin\psi)'\Sigma^p + \epsilon^{pqr}[X_q, X_r]/2\Big)\,\epsilon_0\Big)\\ &= 0\end{aligned} \tag{2.27}$$

where $\varepsilon^{456} = 1,\ \varepsilon^{789} = 1,\ 2\Sigma^a = -\varepsilon^{abc}\Gamma_{bc},\ 2\Sigma^p = -\varepsilon^{pqr}\Gamma_{qr}$.

The Majorana-Weyl spinor in 16 of SO(1, 9) can be decomposed as $V_8 \otimes V_2$, where V_8 transforms in the irreducible representation $2 \otimes 2_X \otimes 2_Y$ of $\mathrm{SO}(1,2)\times \mathrm{SO}(3)_X \times \mathrm{SO}(3)_Y$ and V_2 is the space acted by $\mathrm{SL}(2,\mathbb{R})$ which is generated by $B_0,\ B_1,\ B_2$. The Σ^a are three generators of $\mathrm{SO}(3)_X$ which acts on the space 2_X and Σ^p are three generators of $\mathrm{SO}(3)_Y$ which acts on the space 2_Y. The fifteen matrixes $\Sigma^a,\ \Sigma^p,\ \Sigma^a\Sigma^p$ are anti-Hermitian and traceless acting on $2_X \otimes 2_Y$. As the trace of the product of arbitrary two different matrixes vanishes, they are independent matrixes acting on $2_X \otimes 2_Y$. And since $B_2\epsilon_0 = \epsilon_0$ and $B_2B_0\epsilon_0 = -\epsilon_0$, so ϵ_0 and $B_0\epsilon_0$ are two independent vectors in the space V_2. Therefore the vanishing of the gaugino supersymmetry transformation leads to

$$\begin{aligned}(X_a\cos\psi)' = 0,&\qquad (X_p\sin\psi)' = 0\\ [X_a, X_b] = 0,&\qquad [X_p, X_q] = 0,\quad [X_a, X_p] = 0,\end{aligned} \tag{2.28}$$

with the solution

$$X_a = \frac{C_a}{\cos\psi},\qquad X_p = \frac{D_p}{\sin\psi}, \tag{2.29}$$

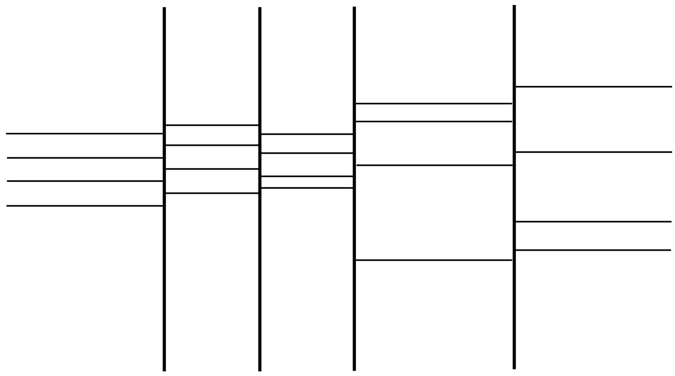

Figure 1: A system of N parallel D3-branes intersecting successive NS5-branes. A generalized Janus configuration with an arbitrary y-dependence of the gauge coupling can be obtained as a limit of this.

where C_a, D_p's are constant matrices commuting with each other. This is the only vacuum we can have. The brane configuration is picture 1. Like the case of $\mathcal{N} = \triangle$ SYM, the constant C_a, D_p's characterize the transverse positions of D3-branes.

One can also study the vacuum directly from the energy functional

$$\begin{aligned} H = & \int d^3x \frac{1}{e^2} \mathrm{Tr}((\sin\psi \overline{F}_{34} - F_{56})^2 + (\sin\psi \overline{F}_{35} - F_{64})^2 + (\sin\psi \overline{F}_{36} - F_{45})^2 \\ & + (\cos\psi \overline{F}_{37} - F_{89})^2 + (\cos\psi \overline{F}_{38} - F_{97})^2 + (\cos\psi \overline{F}_{39} - F_{78})^2 \\ & + F_{ap} F^{ap}) + \left(\frac{2}{e^2} \sin\psi \mathrm{Tr}(X_4[X_5, X_6]) + \frac{2}{e^2} \cos\psi \mathrm{Tr}(X_7[X_8, X_9]) \right)' \\ & + \mathrm{Tr} \left(\frac{1}{e^2} \psi' \tan\psi X_a X^a - \frac{1}{e^2} \psi' \cot\psi X_p X^p \right)'. \end{aligned} \tag{2.30}$$

The energy is bounded by the boundary term. If the boundary term vanishes, the classical vacuum configurations satisfy the above equations (2.28) which we obtained from full supersymmetry conditions of vacuum configurations. Thus we can conclude that there is no nontrivial vacuum for Janus configuration with theta term. This is quite different from the case without theta term, which has a nontrivial vacuum characterized by a Nahm-like equation.

3. BPS solutions and brane configurations

In this section, we try to obtain the BPS solutions which preserve part of the supersymmetries in generalized Janus configuration. This require ϵ_0 to satisfy extra projection conditions which are also compatible with the original condition $B_2\epsilon_0 = \epsilon_0$. The possible projection conditions for supersymmetric parameter ϵ_0 are:

$$\Gamma_{123p}\epsilon_0 = \alpha\epsilon_0, \quad \Gamma^{0a}\epsilon_0 = \beta\epsilon_0 \tag{3.1}$$

where $\alpha = \pm 1, \beta = \pm 1$. Without losing generality, we just set $p = 7, a = 4$ in the following discussion. In this case we have the following identities:

$$\Gamma_{1289}\epsilon_0 = -\alpha\epsilon_0, \quad \Gamma_{1256}\epsilon_0 = \beta\epsilon_0, \quad \Gamma_{3567}\epsilon_0 = -\alpha\beta\epsilon_0, \quad \Gamma_{5689}\epsilon_0 = \alpha\beta\epsilon_0 \tag{3.2}$$

$$B_0\epsilon_0 = -\Gamma_{0123}\epsilon_0 = \beta\Gamma_{1234}\epsilon_0 = -\Gamma_{3456}\epsilon_0 = -\alpha\Gamma_{07}\epsilon_0 = -\alpha\beta\Gamma_{47}\epsilon_0 \tag{3.3}$$

$$= \alpha\beta\Gamma_{3489}\epsilon_0 = -\alpha\Gamma_{0389}\epsilon_0 = \beta\Gamma_{0356}\epsilon_0 \tag{3.4}$$

If we impose one projection condition in (3.1) then we get 1/2 BPS configurations. If imposing both conditions we obtain 1/4 BPS configurations. After multiplying a factor $-(\cos\frac{\psi}{2} + \sin\frac{\psi}{2}B_0)$, the supersymmetry transformation of the gaugino field becomes

$$\begin{aligned}
&\Gamma^{12}(F_{12} - F_{56}\Gamma_{1256} - F_{89}\Gamma_{1289} + \sin\psi\overline{F}_{34}\Gamma_{1234} - \overline{F}_{37}\cos\psi\Gamma_{1237})\epsilon_0 \\
&+\Gamma^{23}(F_{23} - \cos\psi F_{17}\Gamma_{1237} + \sin\psi F_{14}\Gamma_{1234}B_0)\epsilon_0 \\
&+\Gamma^{31}(\cos\psi F_{31} - \cos\psi F_{27}\Gamma_{1237} + \sin\psi F_{24}\Gamma_{1234}B_0)\epsilon_0 \\
&+(\cos\psi + \sin\psi B_0)\left[\Gamma^{15}(F_{15} + F_{26}\Gamma_{1256})\right. \\
&\left.+\Gamma^{16}(F_{16} - F_{25}\Gamma_{1256}) + \Gamma^{18}(F_{18} + F_{29}\Gamma_{1289}) + \Gamma^{19}(F_{19} - F_{28}\Gamma_{1289})\right]\epsilon_0 \\
&+\Gamma^{01}(F_{01} + \cos\psi F_{14}\Gamma^{04} + \sin\psi F_{17}\Gamma_{07}B_0)\epsilon_0 + \Gamma^{02}(F_{02} + \cos\psi F_{24}\Gamma^{04} \\
&+\sin\psi F_{27}\Gamma_{07}B_0)\epsilon_0 + \Gamma^{03}(F_{03} + \cos\psi\overline{F}_{34}\Gamma^{04} + \sin\psi\overline{F}_{37}\Gamma_{07}B_0)\epsilon_0 \\
&+\Gamma^{04}(\cos\psi F_{04} - \sin\psi F_{07}\Gamma_{47}B_0)\epsilon_0 + \Gamma^{05}(\cos\psi F_{05} - F_{45}\Gamma^{04} \\
&-\sin\psi\overline{F}_{36}\Gamma_{0356}B_0)\epsilon_0 + \Gamma^{06}(\cos\psi F_{06} - F_{46}\Gamma^{04} + \sin\psi\overline{F}_{35}\Gamma_{0356}B_0)\epsilon_0 \\
&+\Gamma^{07}(\cos\psi F_{07} - F_{47}\Gamma^{04} + \sin\psi F_{04}\Gamma_{47}B_0)\epsilon_0 + \Gamma^{08}(\cos\psi F_{08} - F_{48}\Gamma^{04} \\
&-\sin\psi\overline{F}_{39}\Gamma_{0389}B_0)\epsilon_0 + \Gamma^{09}(\cos\psi F_{09} - F_{49}\Gamma^{04} + \sin\psi\overline{F}_{38}\Gamma_{0389}B_0)\epsilon_0 \\
&+\Gamma^{58}(F_{58} + F_{69}\Gamma_{5689})\epsilon_0 + \Gamma^{59}(F_{59} - F_{68}\Gamma_{5689})\epsilon_0 + \Gamma^{35}(\cos\psi\overline{F}_{35} - F_{67}\Gamma_{3567} \\
&-\sin\psi F_{06}\Gamma_{0356}B_0)\epsilon_0 + \Gamma^{36}(\cos\psi\overline{F}_{36} + F_{57}\Gamma_{3567} + \sin\psi F_{05}\Gamma_{0356}B_0)\epsilon_0 \\
&+\Gamma^{38}(\cos\psi\overline{F}_{38} + F_{79}\Gamma^{3789} - \sin\psi F_{09}\Gamma_{0389}B_0)\epsilon_0 \\
&+\Gamma^{39}(\cos\psi\overline{F}_{39} - F_{78}\Gamma^{3789} + \sin\psi F_{08}\Gamma_{0389}B_0)\epsilon_0
\end{aligned}$$

The gaugino transformation should vanish for BPS configurations. Imposing projection condition (3.1), $\delta\Psi = 0$ if all terms vanish separately. This leads to the following nontrivial part of 1/4 BPS equations:

$$\begin{aligned}
F_{12} - \beta F_{56} + \alpha F_{89} + \beta\sin\psi\overline{F}_{34} - \alpha\cos\psi\overline{F}_{37} &= 0 \\
F_{23} + \beta\sin\psi F_{14} - \alpha\cos\psi F_{17} = 0, \qquad F_{31} + \beta\sin\psi F_{24} - \alpha\cos\psi F_{27} &= 0 \\
F_{18} - \alpha F_{29} = 0, \quad F_{19} + \alpha F_{28} &= 0 \\
F_{15} + \beta F_{26} = 0, \quad F_{16} - \beta F_{25} = 0, \quad F_{59} - \alpha\beta F_{68} &= 0 \\
F_{58} + \alpha\beta F_{69} = 0, \qquad F_{01} + \alpha\sin\psi F_{17} + \beta\cos\psi F_{14} &= 0 \\
F_{02} + \alpha\sin\psi F_{27} + \beta\cos\psi F_{24} = 0, \qquad F_{03} + \alpha\sin\psi\overline{F}_{37} + \beta\cos\psi\overline{F}_{34} &= 0 \\
\cos\psi F_{04} + \alpha\beta\sin\psi F_{07} = 0, \qquad \cos\psi F_{05} - \beta F_{45} + \beta\sin\psi\overline{F}_{36} &= 0 \\
\cos\psi F_{06} - \beta F_{46} - \beta\sin\psi\overline{F}_{35} = 0, \qquad \cos\psi F_{07} - \beta F_{47} - \alpha\beta\sin\psi F_{04} &= 0
\end{aligned}$$

$$
\begin{aligned}
\cos\psi F_{08}-\beta F_{48}-\alpha\sin\psi\overline{F}_{39}=0,\qquad \cos\psi F_{09}-\beta F_{49}+\alpha\sin\psi\overline{F}_{38}&=0\\
\cos\psi\overline{F}_{35}+\beta\sin\psi F_{06}+\alpha\beta F_{67}=0,\quad \cos\psi\overline{F}_{36}-\beta\sin\psi F_{05}-\alpha\beta F_{57}&=0\\
\cos\psi\overline{F}_{38}-\alpha\sin\psi F_{09}+F_{79}=0,\qquad \cos\psi\overline{F}_{39}+\alpha\sin\psi F_{08}-F_{78}&=0
\end{aligned}
\tag{3.5}
$$

These equations contain unknown parameter ψ which depends on y, and seem to be impossible to solve.

It is easier to deal with half BPS configurations. The nontrivial part of half BPS equation with one of the projection conditions $\Gamma_{1237}\epsilon_0=\alpha\epsilon_0$ and $\beta=0$ is made of

$$
\begin{aligned}
F_{12}+\alpha F_{89}-\alpha\cos\psi\overline{F}_{37}&=0, & F_{23}-\alpha\cos\psi F_{17}&=0\\
F_{31}-\alpha\cos\psi F_{27}&=0, & F_{01}+\alpha\sin\psi F_{17}&=0\\
F_{02}+\alpha\sin\psi F_{27}&=0, & F_{03}+\alpha\sin\psi\overline{F}_{37}&=0\\
F_{18}-\alpha F_{29}&=0, & F_{19}+\alpha F_{28}&=0\\
\cos\psi F_{08}-\alpha\sin\psi\overline{F}_{39}&=0, & \cos\psi\overline{F}_{38}-\alpha\sin\psi F_{09}+F_{79}&=0\\
\cos\psi F_{09}+\alpha\sin\psi\overline{F}_{38}&=0, & \cos\psi\overline{F}_{39}+\alpha\sin\psi F_{08}-F_{78}&=0.
\end{aligned}
\tag{3.6}
$$

The last two lines of the above equations could be reduced to

$$
\overline{F}_{39}=\cos\psi F_{78}\qquad F_{08}=\alpha\sin\psi F_{78} \tag{3.7}
$$
$$
\overline{F}_{38}=-\cos\psi F_{79}\quad F_{09}=\alpha\sin\psi F_{79}. \tag{3.8}
$$

One can simplify the equations further by let $X_8=X_9=0$, then the first six equations are the dyon equation when ψ is a constant. Although generically ψ is not a constant in the Janus field theory, the above equations could be organized as

$$
\begin{aligned}
B_i&=\alpha\cos\psi D_iX_7,\ i=1,2,\qquad B_3=\alpha\cos\psi\frac{D_3\overline{X}_7}{\sin\psi}\\
E_i&=-\alpha\sin\psi D_iX_7,\ i=1,2,\qquad E_3=-\alpha\sin\psi\frac{D_3\overline{X}_7}{\sin\psi}
\end{aligned}
\tag{3.9}
$$

where $\overline{X}_7=\sin\psi X_7,\ E_i=F_{0i},\ B_i=\frac{1}{2}\epsilon^{ijk}F_{jk}$.[2] One may take them as modified equations for dyons, as we will show soon. They are quiet different from the monopole equations in [11]. It would be interesting to solve these equations. We will discuss the solution in the sharp interface case.

The trivial part of the BPS equation involves the equations on $X_{4,5,6}$, which requires them to be constant. For simplicity, we set $X_{4,5,6}=0$.

Let us consider the energy functional in the case that the six adjoint scalars except X_7 vanish. The energy functional takes the following form

$$
\begin{aligned}
H=&-\int d^3x\frac{1}{e^2}\mathrm{Tr}((B_i-\alpha\cot\psi D_i\overline{X}_7)^2+(E_i+\alpha D_i\overline{X}_7)^2)\\
&+\alpha\int d^3x\partial_i\mathrm{Tr}\left(\frac{2}{e^2}(\cot\psi B_i\overline{X}_7-E_i\overline{X}_7)\right)
\end{aligned}
\tag{3.10}
$$

[2]This is notation for the magnetic field strength without confusing with the previous notation for gamma matrix products.

To get it, we have used the Gauss law

$$D_i\left(\frac{1}{e^2}E_i\right)+2\frac{\psi'}{e^2}B_3=0. \tag{3.11}$$

It is obvious that the energy functional is in consistent with the BPS equation we obtained from supersymmetry analysis, if the boundary term is ignored.

Since it is hard to solve the half-BPS equations, it is useful to recall the BPS soliton in usual field theory. For simplicity, let us assume the gauge group to be SU(2). The adjoint scalar is written as ϕ taking vacuum expectation value $\phi^a\phi^a=e^2v^2$ in the spatial infinity where SU(2) is broken to U(1). In the traditional field theory, the coupling is a constant and the magnetic charge Q_m is related to the winding number

$$Q_m=2\int_{s^2_{\rm inf}}dS^i\frac{1}{e^2v}\mathrm{Tr}(\phi B_i)=\frac{4\pi n_m}{e} \tag{3.12}$$

where n_m is the winding number of scalar field configuration. However when the coupling is spatially varying, the relation between magnetic charge and the winding number is not clear. For the electric charge,

$$Q_e=-2\int_{s^2_{\rm inf}}dS^i\mathrm{Tr}\frac{1}{e^2v}E_i\phi, \tag{3.13}$$

we have to take into account of the Witten effect [15] in the presence of the theta term. The generator that generates the gauge transformations around the direction ϕ^a is $\delta A^a_\mu=(1/ev)D_\mu\phi^a$, whose corresponding Noether charge is

$$\begin{aligned}n_e&=\int d^3x\frac{\partial\mathcal{L}}{\partial(\partial_0A^a_\mu)}\delta A^a_\mu\\&=\int_{s^2_{\rm inf}}dS^i\mathrm{Tr}\left(-\frac{2}{e^3v}E_i\phi+\frac{\theta}{4\pi^2ev}B_i\phi\right)\\&=\frac{Q_e}{e}+\frac{\theta e}{8\pi^2}Q_m\end{aligned} \tag{3.14}$$

by using the Gauss Law. n_e must be an integer, since 2π rotation is not a transformation and leaves the state invariant, giving $e^{i2\pi n_e}=1$.

In our case, we can identify X_7 as the scalar field ϕ above. Then using the corresponding BPS equations (3.9) and $1/e^2=D\sin 2\psi,\theta=2\pi(a+4\pi D\cos 2\psi)$, one can find

$$n_e/n_m=a+4\pi D. \tag{3.15}$$

We will show that the similar relation also appears in the discussion of brane configuration of BPS solition as (p,q)-string. This suggest n_e and n_m should be identified as the charges carried by (p,q) string on D3-brane worldvolume.

For Janus configuration with spatially varying coupling and theta term, one has to carefully define the electric and the magnetic charge of the BPS soliton. Let us take the following definition, which could be reduced to the usual ones consistently:

$$Q_m=2\int d^3x\partial_i\mathrm{Tr}\left(\frac{1}{e^2v}B_iX_7\right),\quad Q_e=-2\int d^3x\partial_i\mathrm{Tr}\left(\frac{1}{e^2v}E_iX_7\right). \tag{3.16}$$

The energy of half BPS soliton solution is bounded by

$$H \geqslant \left| \int d^3x \partial_i \mathrm{Tr}\left(\frac{2}{e^2}(\cos\psi B_i X_7 - \sin\psi E_i X_7)\right)\right| \tag{3.17}$$

with equality if and only if the BPS equations are obeyed. This inequality looks different from the usual BPS relation between the mass and the charges. However, if we consider a simple case with ψ being a constant, then we have

$$\begin{aligned} M &\geq v|\cos\psi Q_m + \sin\psi Q_e| \\ &\geq v\sqrt{Q_m^2 + Q_e^2}. \end{aligned} \tag{3.18}$$

In the above relation the equality is saturated if the BPS equation is satisfied and $\tan\psi = Q_e/Q_m$ which does hold taking into account of the relations (3.14), (3.12), (3.15) and the expression of e^2 and θ in terms of ψ. This suggests that the half BPS solution is actually a dyon. For a dyon with charges (p, q), M is proportional to $|p + q\tau|$ with τ being a complex coupling constant, and

$$\psi + \frac{1}{2}\pi = \arg(p + q\tau). \tag{3.19}$$

This sounds somehow strange since ψ is an arbitrary function in Janus configuration. However, as we will show in the discussion on brane realization of BPS soliton, in order to have half BPS configuration, the possible (p, q) string is very restricted and indeed (3.19) must be respected.

Here seems a puzzle. When ψ being a constant, the Lagrangian of Janus configuration reduces simply to the one of $\mathcal{N} = 4$ super-Yang-Mills theory. And there should be various possible dyonic solutions in the theory. At first sight this is in contradiction with the result we just obtained. However, note that even when ψ being a constant, we have extra projection condition on supersymmetry parameter and the field theory has actually half of the original supersymmetries. The extra projection condition leads to stringent constraints on the possible BPS solitonic solutions. In the brane picture, the extra projection condition comes from the presence of 5-brane system. The constancy of ψ along the whole y corresponds to the case the ψ being the same on two sides of 5-brane system. In this case, the explicit solution of BPS soliton solution is the same as the one in $\mathcal{N} = 4$ SYM with the dyon charges being relatively fixed.

Next let us consider a little more complicated case. We assume that ψ take different values on two sides of 5-brane locating at $y = 0$. In this so-called sharp interface case, ψ changes from one constant value to another one at the interface, namely

$$\psi(y) = \begin{cases} \psi_1, & y > 0 \\ \psi_2, & y < 0. \end{cases} \tag{3.20}$$

Similar to [11] one can still solve the BPS equations in the abelian limit that the nonabelian core size vanishes. In the abelian limit, the question is simplified to the one in traditional electrodynamics. However, one has to be careful since we are discussing the dyon which induce both point-like electric and magnetic sources. In fact, from BPS equation, the fact

$B_i/E_i = -\cot\psi$ suggests that B_i and E_i cannot be both continuous across the interface since ψ is different on two sides. In the following discussion, we just focus on the magnetic field and the electric field is given by $E_i = -\tan\psi B_i$. For a single dyon with one unit of magnetic charge at $y = y_0 > 0$, we have[3]

$$B_i = \cot\psi D_i \overline{X}_7 = i\sigma_3/2 \begin{cases} \frac{(x_1,x_2,y-y_0)}{r_+^3} + \frac{\cot^2\psi_1 - \cot^2\psi_2}{\cot^2\psi_1+\cot^2\psi_2}\frac{(x_1,x_2,y+y_0)}{r_-^3}, & y>0 \\ \frac{2\cot^2\psi_2}{\cot^2\psi_1+\cot^2\psi_2}\frac{(x_1,x_2,y-y_0)}{r_+^3}, & y<0 \end{cases} \tag{3.21}$$

where $r_\pm = x_1^2 + x_2^2 + (y \mp y_0)^2$. The scalar field is

$$X_7 = i\sigma_3/2 \begin{cases} \frac{1}{\sin\psi_1}(\tilde{v} - \frac{1}{\cot^2\psi_1 r_+} - \frac{\cot^2\psi_1-\cot^2\psi_2}{\cot^2\psi_1(\cot^2\psi_1+\cot^2\psi_2)}\frac{1}{r_-}), & y>0 \\ \frac{1}{\sin\psi_2}(\tilde{v} - \frac{2}{\cot^2\psi_1+\cot^2\psi_2}\frac{1}{r_+}), & y<0 \end{cases} \tag{3.22}$$

where $\tilde{v} = \sqrt{\frac{\sin\psi_1}{2D\cos\psi_1}}v$ and v is a constant being related to X_7/e at positive infinity. It is easy to see that the total magnetic flux is 4π at the spacial infinity. And the charges of the dyon are

$$Q_m = \frac{4\pi(\cos\psi_1\cot^2\psi_1 + \cos\psi_2\cot^2\psi_2)}{\cot\psi_1^2 + \cot^2\psi_2}\sqrt{\frac{2D\sin\psi_1}{\cos\psi_1}} \tag{3.23}$$

$$Q_e = \frac{4\pi(\sin\psi_1\cot^2\psi_1 + \sin\psi_2\cot^2\psi_2)}{\cot\psi_1^2 + \cot^2\psi_2}\sqrt{\frac{2D\sin\psi_1}{\cos\psi_1}} \tag{3.24}$$

when $\psi_1 = \psi_2$ we have $Q_m = 4\pi/e$ which is the charge of a single monopole. To obtain the above solution, one needs to take into continuity condition on various fields. Here we take F_{01}, F_{02}, F_{12} and $\overline{X}_7$ to be continuous at the interface. Obviously the scalar field X_7 is not continuous.

Another simplification is to let $A_i = 0,\ A_0 = \overline{X}_7$ and $X_{7,8,9}$ depend only on y. This leads to the following equations

$$F_{89} = \cos\psi\overline{F}_{37} \tag{3.25}$$

$$F_{97} = \frac{1}{\cos\psi}\overline{F}_{38} \tag{3.26}$$

$$F_{78} = \frac{1}{\cos\psi}\overline{F}_{39}. \tag{3.27}$$

One can also obtain the energy functional in this case

$$\begin{aligned} H = & -\int d^3x\frac{1}{e^2}\mathrm{Tr}(F_{89} - \cos\psi\overline{F}_{37})^2 + (F_{03} + \sin\psi\overline{F}_{37})^2 \\ & +(\cos\psi F_{08} - \sin\psi\overline{F}_{39})^2 + (\cos\psi\overline{F}_{38} - \sin\psi F_{09} + F_{79})^2 \\ & +(\cos\psi F_{09} + \sin\psi\overline{F}_{38})^2 + (\cos\psi\overline{F}_{39} + \sin\psi F_{08} - F_{78})^2 \\ & +\left(\frac{1}{e^2}(2\cos\psi\mathrm{Tr}(X_7[X_8, X_9] - \psi'\cot\psi\mathrm{Tr}X_pX^p - 2F_{03}\overline{X}_7))\right)' \end{aligned} \tag{3.28}$$

[3] For a realistic dyon with charge (p, q), the electric and magnetic field strength is simply the multiple of the ones for one unit charge in the Abelian limit. Therefore, we just focus on the case with one unit charge.

by using the following equation of motion

$$D_3\left(\frac{1}{e^2}F_{03}\right)+\left[X_8,\frac{1}{e^2}F_{08}\right]+\left[X_9,\frac{1}{e^2}F_{09}\right]=0. \tag{3.29}$$

Similarly it is in consistent with the BPS equations. One can also obtain the 1/4 BPS equation from the energy functional analysis which is similar to the above case. It contains some boundary terms and square terms which are left hand of the equations in (3.5).

To solve the BPS equations, without losing generality, we can assume the gauge group to be SU(2) and make the following ansatz,

$$X_7=-if_1(y)\sigma_1/2, \quad X_8=-if_2(y)\sigma_2/2, \quad X_9=-if_3(y)\sigma_3/2, \tag{3.30}$$

with σ_i's being Pauli matrices. The above set of equations could be reduced to

$$f_2f_3=\frac{\cos\psi\partial_y(f_1\sin\psi)}{\sin\psi}, \quad f_1f_3=\frac{\cos\psi\partial_y(f_2\sin\psi)}{\sin\psi}, \quad f_1f_2=\frac{\partial_y(f_3\sin\psi)}{\cos\psi\sin\psi}. \tag{3.31}$$

If ψ is a constant, the above equation could be solved by a proper rescaling of $f_3(y)$. The solutions are

$$\begin{aligned}
f_1(y;k,F,y_0) &= -\frac{F\,cn_k(F(y-y_0))}{sn_k(F(y-y_0))}\\
f_2(y;k,F,y_0) &= -\frac{F\,dn_k(F(y-y_0))}{sn_k(F(y-y_0))}\\
f_3(y;k,F,y_0) &= -\frac{F\cos\psi}{sn_k(F(y-y_0))}
\end{aligned}$$

where sn_k, cn_k, dn_k are Jacobi elliptic functions with k being elliptic modulus and $F\geq 0, y_0$ are arbitrary constants. However if ψ is an arbitrary function of y, the equations (3.27) become very difficult to solve.

Another projection condition $\Gamma^{04}\epsilon_0=\beta\epsilon_0$ and $\alpha=0$ leads to another half BPS configurations

$$\begin{aligned}
F_{12}-\beta F_{56}+\beta\sin\psi\overline{F}_{34}&=0, & F_{23}+\beta\sin\psi F_{14}&=0\\
F_{31}+\beta\sin\psi F_{24}&=0, & F_{01}+\beta\cos\psi F_{14}&=0\\
F_{02}+\beta\cos\psi F_{24}&=0, & F_{03}+\beta\cos\psi\overline{F}_{34}&=0\\
F_{15}+\beta F_{26}&=0, & F_{16}-\beta F_{25}&=0\\
\cos\psi F_{05}-\beta F_{45}+\beta\sin\psi\overline{F}_{36}&=0, & \cos\psi\overline{F}_{35}+\beta\sin\psi F_{06}&=0\\
\cos\psi F_{06}-\beta F_{46}-\beta\sin\psi\overline{F}_{35}&=0, & \cos\psi\overline{F}_{36}-\beta\sin\psi F_{05}&=0.
\end{aligned} \tag{3.32}$$

The discussion is very similar to the former case.

Let us return to the 1/4-BPS equations. Since the projection conditions only involve two directions 4 and 7, one may simplify the discussion by set $X_5=X_6=X_8=X_9=0$

such that the equations could be rewritten as

$$\begin{aligned}
D_iX_4 &= -\beta(\cos\psi E_i + \sin\psi B_i), \quad i=1,2\\
D_iX_7 &= \alpha(-\sin\psi E_i + \cos\psi B_i), \quad i=1,2\\
\frac{D_3(X_4\sin\psi)}{\sin\psi} &= -\beta(\cos\psi E_3 + \sin\psi B_3),\\
\frac{D_3(X_7\sin\psi)}{\sin\psi} &= \alpha(-\sin\psi E_3 + \cos\psi B_3),\\
D_0X_4 &= D_0X_7 = 0, \quad [X_4, X_7] = 0.
\end{aligned} \tag{3.33}$$

When ψ is simply a constant, the above equations are the ones for BPS string junctions in SYM [16, 17]. Therefore the above equations could be taken as the ones for BPS string junction in Janus configuration. It would be interesting to solve these equations.

Next, we will analyze the corresponding brane configurations of the BPS solutions. The brane construction of the theory is D3-brane ending on five-brane. D3-brane extends along the 0123 directions. There may be two groups of five-branes. One of the (p,q) five-brane extends along 012456 directions and the other one (p',q') extends along 012789. If one takes ϵ_1 and ϵ_2 as the supersymmetry parameters of the left and right move modes in type IIB theory, then the unbroken supersymmetry for D3-brane is just

$$\epsilon_2 = \Gamma_{0123}\epsilon_1 \tag{3.34}$$

The ϵ_1 is identical to ϵ in the above Janus field theory. Using (2.20), (2.21), we also have

$$\epsilon_1 = \left(\cos\frac{\psi}{2} - \sin\frac{\psi}{2}B_0\right)\epsilon_0, \quad (\sin\psi B_1 + \cos\psi B_2)\epsilon_1 = \epsilon_1 \tag{3.35}$$

As we have argued before, the projection condition on ϵ_1 can be transformed to the projection condition on ϵ_0. We can write it as $\Gamma'\epsilon_0 = \epsilon_0$. To be compatible with the original projection condition $B_2\epsilon_0 = \epsilon_0$ asks that Γ' and B_2 must commute. But if we want to know the brane configurations of the corresponding BPS configurations, we should know the projection condition on ϵ_1. Let us consider a (p,q)-string extended along 0m directions.

The supersymmetry condition for (p,q)-string is

$$\epsilon_1 = -\Gamma(\cos t\epsilon_1 - \sin t\epsilon_2) \tag{3.36}$$

where $\Gamma = \Gamma_{0m}$ and $t = \arg(q\tau+p)$. According to (3.34), it is equivalent to $\epsilon_1 = -\Gamma(\cos t\epsilon_1 + \sin tB_0\epsilon_1) = -\Gamma e^{tB_0}\epsilon_1$. Using (3.35), we express it in terms of ϵ_0 as follows

$$\epsilon_0 = -e^{\psi B_0/2}\Gamma e^{tB_0}e^{-\psi B_0/2}\epsilon_0. \tag{3.37}$$

If the (p,q)-string extends along 1, 2 or 3 directions, it will be dissolved in D3 branes and form a bound state [18] whose supersymmetry conditions are different from (3.34). In this case, one has to find a noncommutative field theory for Janus configuration [17]. For the (p,q)-string extending along other directions, we have $\Gamma B_0 = -B_0\Gamma$, and then

$$\epsilon_0 = -\Gamma e^{(t-\psi)B_0}\epsilon_0 = -(\cos(t-\psi)\Gamma + \sin(t-\psi)\Gamma B_0)\epsilon_0. \tag{3.38}$$

The compatible condition leads to

$$\cos(t-\psi)=0, \qquad \text{if } \Gamma B_2=-B_2\Gamma \tag{3.39}$$

with the solutions

$$\begin{cases} t=\psi+\pi/2, \ -\Gamma B_0\epsilon_0=\epsilon_0 \\ t=\psi-\pi/2, \ \Gamma B_0\epsilon_0=\epsilon_0 \end{cases} \tag{3.40}$$

or

$$\sin(t-\psi)=0, \qquad \text{if } \Gamma B_2=B_2\Gamma, \tag{3.41}$$

with the solutions

$$\begin{cases} t=\psi, \qquad \Gamma\epsilon_0=-\epsilon_0 \\ t=\psi+\pi, \ \Gamma\epsilon_0=\epsilon_0 \end{cases} \tag{3.42}$$

The relation of t and ψ can determine p/q in terms of a, D or vice versa. For example $t=\psi$ is equivalent to

$$\frac{\sin\psi}{\cos\psi}=\frac{\mathrm{Im}(p+q\tau)}{\mathrm{Re}(p+q\tau)}, \tag{3.43}$$

which leads to $p/q=4\pi D-a$ by using $\tau=a+4\pi De^{2i\psi}$. Similarly $t=\psi+\pi/2$ leads to $p/q=-a-4\pi D$.

The above analysis is for the general case. Now let us be more specific. For (p,q)-string extending along 04 we have $\Gamma=\Gamma_{04}$. Since $\Gamma_{04}B_2=B_2\Gamma_{04}$ satisfying (3.41), we have the corresponding solution (3.42) that $\Gamma^{04}\epsilon_0=\epsilon_0$ for $t=\psi$ or $\Gamma^{04}\epsilon_0=-\epsilon_0$ for $t=\psi+\pi$. The constraint for the charge is

$$p/q=4\pi D-a. \tag{3.44}$$

Here the projection condition is exactly the same one in (3.1) for $a=4$ in generalized Janus configuration. Thus the half BPS solution (3.32) we obtained above could be realized by the brane configuration with (p,q)-string extending along 04. The discussion for (p,q)-strings along 05 and 06 cases is similar.

The thing is actually a little subtler here. In the brane picture, the integration constants a and D are determined by the background branes. For the generalized Janus configuration, one can consistently add 5-branes along 012456 or along 012789 or both [12]. To be consistent with (3.44), generically only 5-branes extending along 012456 are allowable. This brane picture may help us to understand the equations in (3.32). For example, the fact that (p,q)-string along 04 looks like dyon from the point of view of D3-brane extending along 0123 explains the modified dyon equations in (3.32). And the fact that (p,q)-string realize the instantons in transverse directions 1256 of 5-branes is encoded in the fourth line of (3.32).

On the other hand, for the strings along 07 case the projection condition is $\Gamma_{1237}\epsilon_0=\pm\epsilon_0$ as $\Gamma_{07}B_2=-B_2\Gamma_{07}$ and the charges has to satisfy $p/q=-a-4\pi D$, which asks the 5-branes to lie along 012789 consistently. In this case, the parameter $t=\psi+\pi/2$ matches exactly with the relation (49) and the charge ratio is reminiscent of (45). This is exactly in match with the half-BPS solutions in generalized Janus configuration obtained by imposing one of the projection condition in (3.1) with $p=7$. The string orthogonal to D3 branes

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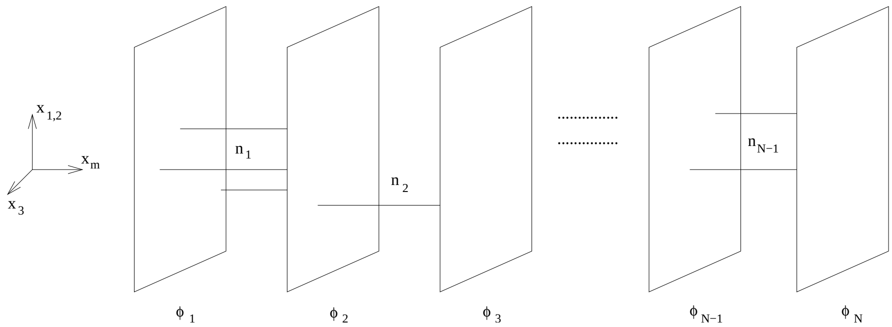

Figure 2: We consider n (p,q)-strings, with worldvolume x_0, x_m, and with D3-brane impurities inserted at particular points $x_m = \phi_a$.

worldvolume realize a dyon in D3 branes. For the strings lie along 08 or 09, we have the same picture.

In the above, to find the BPS brane configurations, we discuss the possibility of adding (p,q)-string configuration in the original Janus brane configuration, without breaking all the supersymmetries. It turns out that to keep half supersymmetry, the strings could lie along 0m, with m being one of transverse directions. The string can come all the way from the infinity and end on the D3-branes. From the D3-brane point of view, the strings looks like a dyon. However this kind of configuration corresponds to the spiky string [19] in the presence of 5-branes instead of the BPS solutions we discussed in generalized Janus configuration. In our case, the right brane configuration is that there is one D3-brane siting far away from other N D3-branes along the direction which the string lie. This corresponds to Higgs mechanism which break SU($N+1$) to SU(N) $\times$ U(1) and give the mass to the corresponding scalar fields. Generically, the D3-branes can sit at different places along direction x^m such that the original gauge group is further broken. The corresponding brane configuration (see [20]) is shown in figure 2.

4. Less supersymmetry Janus configurations with theta angle

Less supersymmetry Janus Yang-Mills theory without theta angle have been considered in [9, 11]. We can use the similar technique to investigate the case with theta angle. We can impose additional project condition to the susy parameter ϵ_0 as long as the conditions are all consistent. In order to make the expression simple, we use the following notation:

$$B_{01} = \Gamma_{3456}, \qquad B_{11} = \Gamma_{3489}, \qquad B_{21} = \Gamma_{3597}, \qquad B_{31} = \Gamma_{3678}, \tag{4.1}$$

$$B_{i2} = B_0 B_{i1}, \qquad i = 0,1,2,3 \tag{4.2}$$

They satisfy the relations $B_0^2 = -1, B_{i1}^2 = B_{i2}^2 = 1, B_0 B_{i1} = -B_{i1} B_0$. The projection conditions for 1/4 supersymmetry configuration which have only two supercharges are

$$B_{i2}\epsilon_0 = h_i \epsilon_0, \tag{4.3}$$

where $h_i = \pm 1, i = 0,1,2,3$. The compatible conditions require $h_0h_1h_2h_3 = -1$. In fact there are only three independent constraints. They are equivalent to

$$(\sin\psi B_{i1} + \cos\psi B_{i2})\epsilon = h_i\epsilon. \tag{4.4}$$

Similar to [12] we can also analyze the brane configuration of the fivebranes that lead to the above supersymmetries. For (p,q)-fivebrane extending along the 012456 directions imposes a constraint

$$\epsilon_1 = -\Gamma_{012456}(\sin t\epsilon_1 + \cos t\epsilon_2). \tag{4.5}$$

If existing both types of branes, then we have

$$\epsilon_1 = -(-\cos tB_{11} + \sin tB_{12})\epsilon_1, \tag{4.6}$$

where $t = \arg(q + p\tau)$. Compared with (4.4), we find $t = \psi \pm \pi/2$. It leads to a constraint on the charges that is $q/p = -a - 4\pi D$. Other projection conditions correspond to (p,q)-fivebranes extended along 012489, 012597 or 012678. Their constraint on charges are the same so that they have the same charge for fivebrane extended along these directions. However the fivebrane also can extend along 012789, 012567, 012648 or 012459 and they have the requirement $t = \psi$ or $t = \psi + \pi$ which lead to the same condition of the charge $q/p = -a + 4\pi D$.

The perturbed supersymmetry transformation could be

$$\delta_1\Psi = \frac{-1}{2}C_i(\Gamma^{a_i}X_{a_i}(s_{i1}\Gamma_3B_{i1} + s_{i2}\Gamma_3B_{i2}) + \Gamma^{p_i}X_{p_i}(t_{i1}\Gamma_3B_{i1} + t_{i2}\Gamma_3B_{i2}))\epsilon, \tag{4.7}$$

where $a_0 = 4,5,6, a_1 = 4,8,9, a_2 = 5,9,7, a_3 = 6,7,8, p_0 = 7,8,9, p_1 = 5,6,7, p_2 = 6,4,8, p_3 = 4,5,9$ and C_i's are constants. The ansatz for perturbed action is

$$I' = \int d^4x \frac{i}{e^2}\mathrm{Tr}\overline{\Psi}\left(\alpha\Gamma_{012} + C_i(\beta_i\Gamma_3B_{i1} + \gamma_i\Gamma_3B_{i2})\right)\Psi, \tag{4.8}$$

$$\begin{aligned} I'' &= \int d^4x \frac{1}{e^2}\Bigg(u\varepsilon^{\mu\nu\lambda}\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda\right) \\ &\quad + \sum_{i=0}^{3}\left(C_i\frac{v_i}{3}\varepsilon^{a_ib_ic_i}\mathrm{Tr}X_{a_i}[X_{b_i}, X_{c_i}] + C_i\frac{w_i}{3}\varepsilon^{p_iq_ir_i}\mathrm{Tr}X_{p_i}[X_{q_i}, X_{r_i}]\right)\Bigg), \end{aligned} \tag{4.9}$$

$$I''' = \int d^4x \mathrm{Tr}(\frac{1}{e^2}r_{mn}X_mX_n), \tag{4.10}$$

where ε are antisymmetric tensors normalized as $\varepsilon^{456} = \varepsilon^{489} = \varepsilon^{597} = \varepsilon^{678} = \varepsilon^{789} = \varepsilon^{567} = \varepsilon^{648} = \varepsilon^{459} = 1$. There are undetermined parameters $\alpha, \beta_i, \gamma_i, u, v_i, w_i, r_{mn}$ in the perturbed action, which should be determined by supersymmetry. Since the zeroth order and first order terms under supersymmetry variation are linear in C_0, C_1, C_2, C_3, the terms containing C_i vanish separately. Then the equations for the parameters are similar to (2.10)–(2.14), where $B_1, B_2, \beta, \gamma, v, w$ become $B_{i1}, B_{i2}, \beta_i, \gamma_i, v_i, w_i$. So we have similar solutions for the undetermined parameters

$$\psi' = 2\alpha, \qquad \beta_i = -h_i \frac{\psi'}{2\cos\psi}, \qquad \gamma_i = h_i \frac{\psi'}{2\sin\psi}, \tag{4.11}$$

$$u = -4\alpha, \qquad v_i = -4\beta_i, \qquad w_i = -4\gamma_i, \qquad \frac{1}{e^2} = D\sin 2\psi, \tag{4.12}$$

$$s_{i1} = 2h_i\psi' \frac{\sin^2\psi}{\cos\psi}, \qquad s_{i2} = 2h_i\psi'\sin\psi, \tag{4.13}$$

$$t_{i1} = -2h_i\psi'\cos\psi, \qquad t_{i2} = -2h_i\psi' \frac{\cos^2\psi}{\sin\psi}. \tag{4.14}$$

However there is an additional constraint for C_i that is $\sum_i C_i = 1$.

The second order variations of the action come from three parts. The first part is the perturbed modified supersymmetry transformation of I', which will be denoted as $\delta_1 I'$. The second one is from $\delta_1 I$ which contains both the first order and the second order variation parts. We denote the second order variation part as $\delta_1 I|_2$. The last one is from the unperturbed supersymmetry variation of I'''. Their expressions are the following

$$\begin{aligned}
\delta_1 I' &= -i \int d^4x \frac{1}{e^2} \mathrm{Tr}\bar{\epsilon} C_i \left((s_{i1}B_{i1} + s_{i2}B_{i2})\Gamma^{a_i} X_{a_i} + (t_{i1}B_{i1} + t_{i2}B_{i2})\Gamma^{p_i} X_{p_i} \right) \Psi, \\
\delta_1 I|_2 &= -i \int d^4x \mathrm{Tr} \left(\left(\frac{q}{2e^2} + \frac{1}{e^2}\frac{\mathrm{d}}{\mathrm{dy}} \right) (\bar{\epsilon} C_i (s_{i1}B_{i1} + s_{i2}B_{i2})) \right) \Gamma^{a_i} X_{a_i} \Psi \\
&\quad -i \int d^4x \mathrm{Tr} \left(\left(\frac{q}{2e^2} + \frac{1}{e^2}\frac{\mathrm{d}}{\mathrm{dy}} \right) (\bar{\epsilon} C_i (t_{i1}B_{i1} + t_{i2}B_{i2})) \right) \Gamma^{p_i} X_{p_i} \Psi, \\
\delta_0 I''' &= \int d^4x \frac{i}{e^2} r_{mn} \bar{\epsilon} (\Gamma_m X_n + \Gamma_n X_m) \Psi.
\end{aligned} \tag{4.15}$$

Using the identities (4.4), (4.13) and (4.14) and requiring the second order variation of action under susy transformation vanish, we can determine the parameter r_{mn} and have the following form of I'''

$$\begin{aligned}
I''' = \frac{1}{2e^2} \int d^4x & \left(2\psi'^2 + (2\psi'\tan\psi)' \right) \mathrm{Tr} \left((C_0 + C_1)X_4^2 + (C_0 + C_2)X_5^2 \right. \\
& \left. + (C_0 + C_3)X_6^2 + (C_2 + C_3)X_7^2 + (C_1 + C_3)X_8^2 + (C_1 + C_2)X_9^2 \right) \\
& + \left(2\psi'^2 - (2\psi'\cot\psi)' \right) \mathrm{Tr} \left((C_2 + C_3)X_4^2 + (C_1 + C_3)X_5^2 \right. \\
& \left. + (C_1 + C_2)X_6^2 + (C_0 + C_1)X_7^2 + (C_0 + C_2)X_8^2 + (C_0 + C_3)X_9^2 \right) \\
& - 2\left(\frac{\psi\prime^2}{\cos^2\psi} + \frac{\psi\prime^2}{\sin^2\psi} \right) \mathrm{Tr}((C_0 + C_1)(C_2 + C_3)(X_4^2 + X_7^2) \\
& + (C_0 + C_2)(C_1 + C_3)(X_5^2 + X_8^2) + (C_0 + C_3)(C_1 + C_2)(X_6^2 + X_9^2)).
\end{aligned} \tag{4.16}$$

The detailed calculation is given in the appendix.

Similar to the case studied in [9, 11], there is enhanced global symmetry SU(3) with 1/4 supersymmetry when $C_0 = C_1 = C_2 = C_3 = 1/4$. For $C_0 = C_1 = 1/2,\ C_2 = C_3 = 0$ the half supersymmetric configuration has enhanced global symmetry $\mathrm{SO}(2) \times \mathrm{SO}(2)$.

With the same method of obtaining eight supercharges vacuum structure, we can analyze the vacuum structure of half supersymmetric configuration. Without making confusion, we take the following notation:

$$\tilde{X}_{i+3} = X_{i+3}(\cos\psi)^{C_0+C_i}(\sin\psi)^{1-C_0-C_i}, \quad \tilde{F}_{3\ i+3} = \frac{D_3\tilde{X}_{i+3}}{(\cos\psi)^{C_0+C_i}(\sin\psi)^{1-C_0-C_i}}, \tag{4.17}$$

$$\tilde{X}_{i+6} = X_{i+6}(\sin\psi)^{C_0+C_i}(\cos\psi)^{1-C_0-C_i}, \quad \tilde{F}_{3\ i+6} = \frac{D_3\tilde{X}_{i+6}}{(\sin\psi)^{C_0+C_i}(\cos\psi)^{1-C_0-C_i}}, \tag{4.18}$$

where $i = 1, 2, 3$. With these notations one can simplify the expression of the part action $I + I'''$. Replacing the terms $\frac{1}{2}F_{3a}F^{3a} + \frac{1}{2}F_{3p}F^{3p}$ in I with $\frac{1}{2}\tilde{F}_{3a}\tilde{F}^{3a} + \frac{1}{2}\tilde{F}_{3p}\tilde{F}^{3p}$ and other terms not changing, the modified action of I is then identical to the original action $I + I'''$ except for additional boundary terms proportional to ψ' which vanishes if $\psi' = 0$ at infinite.

Let us consider the case $C_2 = C_3 = 0, C_0 + C_1 = 1$. Taking the ansatz $X_5 = X_6 = 0, A_\mu = 0, A_3 = 0$, $\Gamma_{3789} = \Gamma_{3567}\epsilon_0 = \epsilon_0$ and the scalars only depending on x_3, we obtain the following equations,

$$\sin\psi\tilde{F}_{34} + \cos\psi\tilde{F}_{37} - F_{89} = 0, \qquad \sin\psi\tilde{F}_{38} + F_{49} = 0 \tag{4.19}$$

$$\sin\psi\tilde{F}_{39} - F_{48} = 0, \qquad \sin\psi\tilde{F}_{37} - \cos\psi\tilde{F}_{34} = 0 \tag{4.20}$$

$$\cos\psi\tilde{F}_{38} + F_{79} = 0, \qquad \cos\psi\tilde{F}_{39} - F_{78} = 0 \tag{4.21}$$

Also we can obtain these equations from the energy functional

$$\begin{aligned} H = & -\int d^3x \frac{1}{e^2}\mathrm{Tr}((\sin\psi\tilde{F}_{37} - \cos\psi\tilde{F}_{34})^2 + (\sin\psi\tilde{F}_{38} + F_{49})^2 + (\sin\psi\tilde{F}_{39} - F_{48})^2 \\ & +(\sin\psi\tilde{F}_{34} + \cos\psi\tilde{F}_{37} - F_{89})^2 + (\cos\psi\tilde{F}_{38} - F_{97})^2 + (\cos\psi\tilde{F}_{39} - F_{78})^2 \\ & + \left(\frac{2}{e^2}\sin\psi\mathrm{Tr}(X_4[X_8, X_9]) + \frac{2}{e^2}\cos\psi\mathrm{Tr}(Y_7[Y_8, Y_9])\right)', \end{aligned} \tag{4.22}$$

where we have omitted a boundary term proportional to ψ' which is vanishing at infinity after integration. The energy is bounded below by the boundary term. When the boundary term vanishes, the minima of the energy gives the equations (4.19)–(4.21). The trivial vacuum is to let all X's commute with each other such that $\tilde{X}_4, \tilde{X}_p$ are just constant. This is similar to the vacuum of generalized Janus configuration. However, for the less supersymmetric case, X_p have different dependence on ψ for different values of C_0, C_1. This means that the vacuum is different for different less susy Janus configurations.

To get the nontrivial solutions of (4.19)–(4.21) seems difficult. Since we do not know the dependence of ψ on y, we can not solve the equation directly. However they look like Nahm equations. From the second equation in (4.20), we have $\tilde{X}_4 = \tilde{X}_7 + c$ with c being a constant. The above equations are simplified as

$$D_3\tilde{X}_7 = (\tan\psi)^{C_1-C_0}[\tilde{X}_8, \tilde{X}_9], \quad \sin\psi\cos\psi D_3\tilde{X}_8 = [\tilde{X}_9, \tilde{X}_7], \quad \sin\psi\cos\psi D_3\tilde{X}_9 = [\tilde{X}_7, \tilde{X}_8]$$

Now let us prove there is no nontrivial vacuum. Without losing generality, we can assume the gauge group to be SU(2) and make the following ansatz,

$$\tilde{X}_7 = -i\sigma_1 g_1, \qquad \tilde{X}_8 = -ig_2\sigma_2, \qquad \tilde{X}_9 = -ig_3\sigma_3 \tag{4.23}$$

$$g_1 = \frac{f_1(y)}{\sin 2\psi}, \qquad g_2 = \frac{f_2(y)}{2\sin\psi^{C_0}\cos\psi^{C_1}}, \qquad g_3 = \frac{f_3(y)}{2\sin\psi^{C_0}\cos\psi^{C_1}} \tag{4.24}$$

with σ_i's being Pauli matrices. If ψ is a constant, the above set of equations could be reduced to

$$f_2 f_3 = \partial_y f_1, \quad f_1 f_3 = \partial_y f_2, \quad f_1 f_2 = \partial_y f_3, \tag{4.25}$$

whose solutions are

$$\begin{aligned} f_1(y;k,F,y_0) &= -\frac{F cn_k(F(y-y_0))}{sn_k(F(y-y_0))} \\ f_2(y;k,F,y_0) &= -\frac{F dn_k(F(y-y_0))}{sn_k(F(y-y_0))} \\ f_3(y;k,F,y_0) &= -\frac{F}{sn_k(F(y-y_0))} \end{aligned} \tag{4.26}$$

where sn_k, cn_k, dn_k are Jacobi elliptic functions with k being elliptic modulus and $F \geq 0, y_0$ are arbitrary constants. The permutations of f_1, f_2 and f_3 are still the solutions of the above equations.

However when generically ψ is not a constant and depends on y, it is not easy to solve the equations. Nevertheless let us start from simple case in which ψ is a constant in the section (y_j, y_{j+1}), where $j = 1, 2, \ldots n$ and $y_j < y_{j+1}$. Namely the y_j's divide the x_3 coordinate into $n+2$ sections and $\psi(y)$ is a ladder function. In different section, ψ, F, k and y_0 can have different values. Note that sn_k is a periodic function with period 4K(k), where K(k) is the complete elliptic integral of the first kind. The function K(k) goes to infinity at $k = 1$. As the zeros of $sn_k(y)$ are $y = 0, 2K(k)$, the above solutions blow up at the zeros of the function $sn_k(F(y-y_0))$. In order to avoid the infinity in each section, one has to carefully choose the parameters such that in each section $sn_k(F(y-y_0))$ is always positive or negative. Recall that $\frac{1}{e^2} = D \sin 2\psi$, so we have $0 < \psi < \pi/2,\ \sin\psi,\ \cos\psi > 0$. Since $\tilde{X}_7, \tilde{X}_8, \tilde{X}_9$ are continuous functions, then g_2, g_3 are always positive or always negative by using (4.24). However, this requirement can not be satisfied. In the sections $y < y_1$ and $y > y_{n+1}$ we must take $k = 1$ in order to avoid the infinity. Then the solution becomes

$$f_3(y;k=1,F,y_0) = -\frac{F\cosh(F(y-y_0))}{\sinh(F(y-y_0))} \tag{4.27}$$

$$f_1(y;k=1,F,y_0) = f_2(y;k=1,F,y_0) = -\frac{F}{\sinh(F(y-y_0))}, \tag{4.28}$$

so we have $g_2(y_1) > 0, g_3(y_1) > 0$ and $g_2(y_{n+1}) < 0, g_3(y_{n+1}) < 0$ which are contrast to the above requirement. This indicates that we can not find nontrivial solutions in the case that ψ is a generalic ladder functions. Since the ladder functions can approach to a general continuous functions, we can conclude that there is no nontrivial vacuum for general profile of $\psi(y)$ satisfying $0 < \psi < \pi/2$. In the Janus configuration without theta angle, there is no nontrivial vacuum if there is no point where e^2 vanishes [11]. In the point where e^2 vanishes, the rescaling scalar need not to be continuous so that there can exist nontrivial solution. However we do not have such special point that the rescaling scalars can not be continuous in the Janus configuration with the theta angle. So finally we do not have nontrivial vacuum.

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Finally, we give the equations for the vacuum preserving two supercharge, corresponding to the case with $C_0 = C_1 = C_2 = C_3 = 1/4$,

$$\begin{aligned}
\sin\psi\tilde{F}_{34} - F_{56} - F_{89} + \cos\psi\tilde{F}_{37} &= 0, \\
\sin\psi\tilde{F}_{35} - F_{64} - F_{97} + \cos\psi\tilde{F}_{38} &= 0, \\
\sin\psi\tilde{F}_{36} - F_{45} - F_{78} - \cos\psi\tilde{F}_{39} &= 0, \\
\sin\psi\tilde{F}_{37} - F_{59} - F_{68} - \cos\psi\tilde{F}_{34} &= 0, \\
\sin\psi\tilde{F}_{38} + F_{49} + F_{67} - \cos\psi\tilde{F}_{35} &= 0, \\
\sin\psi\tilde{F}_{39} - F_{48} + F_{57} + \cos\psi\tilde{F}_{36} &= 0.
\end{aligned}$$

5. Conclusion

In this paper we studied several aspects of Janus configurations with θ-angle. We discussed the vacuum structure of the original field theory proposed in [12], both from the supersymmetry analysis and energy functional. It turned out that the vacuum structure is quite different from the one of Janus configurations studied in [11], where a nontrivial vacuum structure had been discovered. We also investigated the BPS solutions of generalized Janus configurations. These BPS solutions turns out to be the dyons in the field theory, with a nice brane configuration as (p, q)-strings ending on D3-branes. Finally, we discussed the less supersymmetric Janus configurations with θ-angle and proved that it had no nontrivial vacuum. We started from the most general projection conditions and obtained the Janus configurations with two supercharges. We found that in special cases the global symmetry got enhanced and the configurations had more supersymmetries.

We tried to solve the half BPS soliton solutions in the Abelian limit in the sharp interface case. It would be important to find the solutions without taking the abelian limit. And it is also interesting to find 1/4 BPS string-junction solutions. For the case with more interfaces, the construction of the solution is more complicated.

In the study of the brane configurations corresponding to the BPS solutions of generalized Janus configuration, we studied the compatible ways to introduce (p, q)-string in (D3, 5)-brane system. One situation we did not discuss is that when (p,q)-string lie in the worldvolume of D3-brane, in which case the (p,q)-string and D3-brane would form bound state [18]. This would result in a noncommutative Janus configuration. It is interesting to construct such a field theory [21].

In [6 − 8, 22], the half-BPS Janus supergravity solutions were studied systematically. In this case, the global symmetry $OSP(4|4)$ is essential to making ansatz to solve the supergravity equation. For the less supersymmetric case, the global symmetry is further broken due to the presence of other 5-branes. It would be interesting to construct the corresponding supergravity solution [23].

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A. Second order variation of the action (4.8)–(4.10)

The second order variation of action can be simplified as following by using (4.13), (4.14)

$$\begin{aligned}\delta_1 I' &= -i\int d^4x\frac{1}{e^2}\mathrm{Tr}\bar\epsilon(C_i 2\psi'\tan\psi\Gamma^{a_i}X_{a_i} - C_i 2\psi'\cot\psi\Gamma^{p_i}X_{p_i})\\ &\left(\frac{1}{2}\psi' B_0 + C_j h_j\frac{\psi'}{\sin 2\psi}(-\sin\psi B_{j1}+\cos\psi B_{j2})\right)\Psi\\ &= -i\int d^4x\frac{1}{e^2}\mathrm{Tr}\left(\bar\epsilon(-C_i\psi'^2\tan\psi B_0\Gamma^{a_i}X_{a_i} + C_i\psi'^2\cot\psi B_0\Gamma^{p_i}X_{p_i})\right.\\ &+C_iC_jh_j\frac{\psi'^2}{\cos^2\psi}\epsilon\Gamma^{a_i}X_{a_i}(-\sin\psi B_{j1}+\cos\psi B_{j2})\\ &\left.-C_iC_jh_j\frac{\psi'^2}{\sin^2\psi}\epsilon\Gamma^{p_i}X_{p_i}(-\sin\psi B_{j1}+\cos\psi B_{j2})\right)\Psi\end{aligned}\tag{A.1}$$

and

$$\begin{aligned}\delta_1 I|_2 &= -i\int d^4x\mathrm{Tr}\left(\left(\frac{q}{2e^2}+\frac{1}{e^2}\frac{\mathrm{d}}{\mathrm{d}y}\right)(\bar\epsilon C_i 2\psi'\tan\psi)\right)\Gamma^{a_i}X_{a_i}\Psi\\ &+i\int d^4x\mathrm{Tr}\left(\left(\frac{q}{2e^2}+\frac{1}{e^2}\frac{\mathrm{d}}{\mathrm{d}y}\right)(\bar\epsilon C_i 2\psi'\cot\psi)\right)\Gamma^{p_i}X_{p_i}\Psi\\ &= -\frac{i}{e^2}\int d^4x C_i\left(q\psi'\tan\psi + (2\psi'\tan\psi)'\right)\bar\epsilon\Gamma^{a_i}X_{a_i}\Psi\\ &+C_i(\psi'^2\tan\psi)\bar\epsilon B_0\Gamma^{a_i}X_{a_i}\Psi\\ &+C_i\left(-q\psi'\cot\psi - (2\psi'\cot\psi)'\right)\bar\epsilon\Gamma^{p_i}X_{p_i}\Psi\\ &+C_i(-\psi'^2\cot\psi)\bar\epsilon B_0\Gamma^{p_i}X_{p_i}.\Psi\end{aligned}\tag{A.2}$$

Note that $\bar\epsilon\Gamma^{a_i}X_{a_i}(-\sin\psi B_{j1}+\cos\psi B_{j2})$ are proportion to $\bar\epsilon(\sin\psi B_{j1}+\cos\psi B_{j2})\Gamma^{a_i}X_{a_i}$, using the project condition (4.4), they are proportion to $\bar\epsilon h_j\Gamma^{a_i}X_{a_i}$ which is the same form of (4.15). Since susy require $\delta I''' + \delta_1 I' + \delta_1 I|_2 = 0$, such terms should cancel each other. However, we have another form $\bar\epsilon B_0\Gamma^{a_i}X_{a_i}$. These terms cancel each other in (A.1)(A.2).

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